Math 20550 - Calculus III - Summer 2014
July 26, 2014
Final Exam

Name: ____

There is no need to use calculators on this exam. This exam consists of 13 problems on 14 pages. You have 100 minutes to work on the exam. There are a total of 150 available points on the exam. All electronic devices should be turned off and put away. The only things you are allowed to have are: a writing utensil(s) (pencil preferred), an eraser, and an exam. No notes, books, or any other kind of aid are allowed (except your notecard). All answers should be given as exact, closed form numbers as opposed to decimal approximations (i.e., π as opposed to 3.14159265358979...). **You must show all of your work to receive credit. Please box your final answers.** Cheating is strictly forbidden. Good luck!

Honor Pledge: As a member of the Notre Dame community, I will not participate in, nor tolorate academic dishonesty. My signature here binds me to the Notre Dame Honor Code:

Signature:____

Problem	Score
1	/10
2	/10
3	/15
4	/15
5	/10
6	/10
7	/10
8	/10
9	/10
10	/10
11	/15
12	/10
13	/15
Score	/150

Problem 1 (10 points)**.** *Find the Jacobian of the transformation*

$$x = \frac{u}{v}, \quad y = \frac{v}{w}, \quad z = \frac{w}{u}.$$

Problem 2 (10 points). *Is the vector field*

$$\mathbf{F}(x, y) = (3x^2 - 2y^2)\mathbf{i} + (4xy + 3)\mathbf{j}$$

conservative? If so, find a potential function.

Problem 3 (15 points)**.** *Compute* $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$ *where* S *is the sphere of radius* 16*, oriented outward, and* $\mathbf{F}(x,y,z) = \langle xyz, x^2y^2z^2, x^3y^3z^3 \rangle$.

Problem 4 (15 points)**.** *Compute the flux of* $\mathbf{F} = xye^z\mathbf{i} + xy^2z^3\mathbf{j} - ye^z\mathbf{k}$ *through the box bounded by the coordinate planes and the planes* $x = 3$*,* $y = 2$*, and* $z = 1$*, where the box has outward orientation.*

Problem 5 (10 points)**.** *Find and classify all critical points of*

$$f(x, y) = xy - 2x - 2y - x^2 - y^2.$$

Problem 6 (10 points)**.** *Give a vector function which represents the curve of intersection between the hyperboloid* $z = x^2 - y^2$ *and the cylinder* $x^2 + y^2 = 1$.

Problem 7 (10 points)**.** *Is there a vector field* $\mathbf{G}$ *on* $\mathbb{R}^3$ *such that*

$$\text{curl } \mathbf{G} = \left\langle xyz, -y^2z, yz^2 \right\rangle?$$

Explain.

Problem 8 (10 points)**.** *The plane* $2x+y+2z=9$ *intersects the sphere* $x^2+y^2+z^2=9$ *tangentially at exactly one point. Find the point of intersection.*

Problem 9 (10 points)**.** *Find an equation for the sphere which has* $(2, 1, 4)$ *and* $(4, 3, 10)$ *as antipodal points (they are connected by a line though the center of the sphere).*

Problem 10 (10 points)**.** *Determine the value of c such that*

$$g(x,y) = \begin{cases} \dfrac{x^3 + xy^2 + 2x^2 + 2y^2}{x^2 + y^2}, & (x,y) \neq (0,0) \\ \\ c, & (x,y) = (0,0) \end{cases}$$

is continuous.

Problem 11 (15 points)**.** *Compute the line integral* $\int_C \mathbf{F} \cdot d\mathbf{r}$ *where* $\mathbf{F}(x, y) = (y - \cos y)\mathbf{i} + x \sin y\mathbf{j}$ *and* C *is the circle* $(x-3)^2 + (y+4)^2 = 4$*, oriented clockwise. (*Hint: *The area of a circle of radius* r *is* πr^2*.)*

Problem 12 (10 points)**.** *Find* $\dfrac{\partial z}{\partial s}$ *and* $\dfrac{\partial z}{\partial t}$ *where* $z = x^2 - 3y^2$, $x = st$, *and* $y = s + t^2$.

Problem 13 (15 points). *Compute* $\iint_S \text{curl}\mathbf{F} \cdot d\mathbf{S}$*, where* S *is the top and four side faces of the box with vertices* $(\pm 1, \pm 1, \pm 1)$ *(the box without the bottom), given outward orientation, and* $\mathbf{F}(x, y, z) = \langle xyz, xy, x^2yz \rangle$.